Cardiac Motion Estimation by Using Convolutional Sparse Coding

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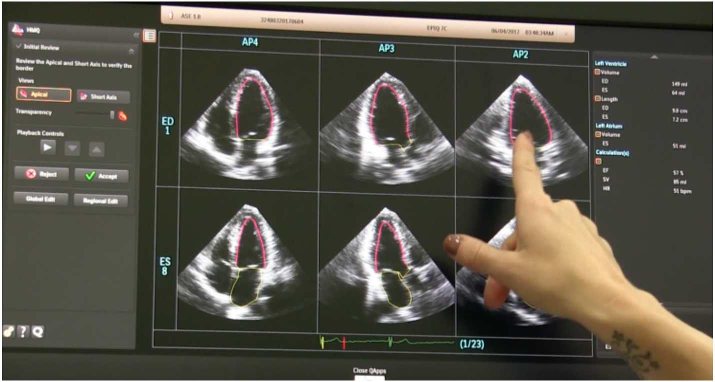
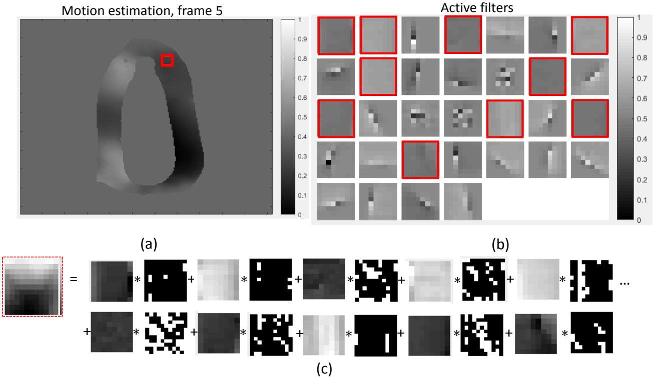
Outline

Motivation

- 50% of deaths are provoked by heart diseases.
- Optimal treatment of cardiac disease requires early detection of abnormalities and accurrate monitoring tools.



Convolutional sparse coding in Ultrasound Imaging



$\mathbf{s}_k$ is modeled as a convolution between the coefficient maps $\mathbf{x}_m$ and a set of M filters $\mathbf{d}_m$ [1],

$$\mathbf{s}_k \approx \sum_{m=1}^{M} \mathbf{d}_m * \mathbf{x}_m \tag{1}$$

Proposed cardiac motion estimation

- A pair of successive frames $(\mathbf{r}_k, \mathbf{r}_{k+1}) \in \mathbb{R}^{J \times N}$ acquired at time instants k and $k+1$
- $(\mathbf{s}_x, \mathbf{s}_y) \in \mathbb{R}^{J \times N}$ where $\mathbf{s}_x$ and $\mathbf{s}_y$ are the motions along the x and y axes.
- Since the motion estimation problem is considered independently the displacement vector is equal to $\mathbf{s} = \mathbf{s}_x$ or $\mathbf{s} = \mathbf{s}_y$.
- The motion estimation field is formulated as the minimization of a cost function with energy $E_{\text{data}}(\mathbf{s})$ penalized by spatial and sparse regularizations, i.e.,

$$\underset{\mathbf{x},\mathbf{s}}{\operatorname{argmin}} \left\{ E_{\text{data}}(\mathbf{s}) + \lambda_d E_{\text{sparse}}(\mathbf{s}, \mathbf{x}) + \lambda_s E_{\text{spatial}}(\mathbf{s}) \right\} \tag{2}$$

Data fidelity

The ML estimator is classically maximize the CPDF in the negative log-domain

$$\underset{\mathbf{s}}{\operatorname{argmin}} - \ln\left[p(\mathbf{r}_{k+1}) | \mathbf{r}_k(n)\right]. \tag{3}$$

Straightforward computations exploiting the Rayleigh statistics of ultrasound imaging detailed in [1] lead to the following data fidelity term

$$E_{\mathrm{data}}(\mathbf{s}) = -2d(\mathbf{s}) + 2\log[e^{2d(\mathbf{s})} + 1] + C \tag{4}$$

where

$$d(\mathbf{s}) = \frac{1}{b}\sum_{n=1}^{N}[\mathbf{r}_{k+1}(n + \mathbf{s}(n)) - \mathbf{r}_k(n)],$$

n indicates the pixel index, $\mathbf{s} = [s(1), \ldots, s(N)]^T$ is the vectorized motion, and $\mathbf{r}_k = [r_k(1), \ldots, r_k(N)]^T$ is the vectorized ultrasound image in frame k, and $C = -\log(2\sigma^4/b)$ is a known constant

Regularization terms

The spatial regularization term promotes the smoothness of the motion estimation field and is defined as

$$E_{\text{spatial}}(\mathbf{s}) = \|\nabla \mathbf{s}\|_2^2 \tag{5}$$

The proposed sparse regularization determines the motion $\mathbf{s}_k$ that is best represented as a convolution between M filters $\mathbf{d}_m$ and the coefficient maps $\mathbf{x}_m$, i.e,

$$E_{\text{sparse}}(\mathbf{s}) = \left\| \mathbf{s}_k - \sum_{m=1}^{M} \mathbf{x}_m * \mathbf{d}_m \right\|_2^2 . \tag{6}$$

Dictionary filters and coefficient maps

A dictionary is estimated off-line by using a set of training cardiac motions denoted as $\widetilde{\mathbf{s}}_k$.

$$\underset{\mathbf{d}_m,\mathbf{x}_{k,m}}{\operatorname{argmin}} \frac{1}{2} \sum_k \left\| \sum_m \mathbf{x}_{k,m} * \mathbf{d}_m - \widetilde{\mathbf{s}}_k \right\|_2^2 + \lambda \sum_m \sum_k \|\mathbf{x}_{k,m}\|_1 \quad (7)$$
$$\text{s.t.} \quad \|\mathbf{d}_m\| = 1 \ \forall m = 1, ..., M$$

The coefficient maps $\mathbf{x}_m$ are computed from test cardiac motions $\hat{\mathbf{s}}_k$

$$\underset{x_m}{\operatorname{argmin}} \frac{1}{2} \left\| \sum_m \mathbf{x}_m * \mathbf{d}_m - \hat{\mathbf{s}}_k \right\|_2^2 + \lambda \sum_m \|\mathbf{x}_m\|_1. \quad (8)$$

Eq (7) and (8) can be solved using alternating direction method of multipliers (ADMM).

Cardiac motion estimation algorithm

Input: $\mathbf{r}_{b,1}, \mathbf{r}_{b,2}, \lambda_s, \lambda_d, K, J, \lambda, \rho,$
$\widetilde{\mathbf{s}}$ = LADdist motions, $\widehat{\mathbf{s}}$ = LADprox motions

Output: $\mathbf{s}$

function $\mathrm{MEFCDL}(\mathbf{r}_{b,1}, \mathbf{r}_{b,2}, \lambda_s, \lambda_d, K, J, \lambda, \rho, \widetilde{\mathbf{s}}, \hat{\mathbf{s}})$
    $\mathbf{d}_m \leftarrow$ Computes the dictionary by solving (7)
    $\mathbf{x}_m \leftarrow$ Computes the coefficient maps by solving (8)
    for $k \leftarrow 1, K$ **do**
        for $j \leftarrow 1, J$ **do**
            $$\operatorname{argmin}_{\mathbf{s}} \{ E_{\mathrm{data}}(\mathbf{r}_{b,1}, \mathbf{r}_{b,2}, \mathbf{s}_{j-1}) + \lambda_s \|\nabla \mathbf{s}_{j-1}\|_2^2 + \lambda_d(k) \|\mathbf{s}_{j-1} - \sum_m \mathbf{d}_m * \mathbf{x}_m\|_2^2 \}$$
            s.t. $\|\mathbf{d}_m\| = 1 \ \forall m$ ▷ Motion estimation
    return $\mathbf{s}$ ▷ (Estimated cardiac motion field)

Experimentation setup

Table 1: Parameters for each step of algorithm 7, dictionary learning, sparse coding, and cardiac motion estimation.

Step	Parameters	Values
Dictionary learning	Database 34 frames	LADdist [2]
	Filter size	16×16
	Filters number	$M = 48$
	Sparsity term	$\lambda = 0.05$
	Number of iteration	500
Sparse coding	Database 34 frames	LADprox [2]
	Number of iteration	500
Cardiac motion estimation	Regularization parameter	$\lambda_s = 0.75$
	Sparsity term (Systole)	$\lambda_d = \{1 \times 10^{-6} \times 10^{-3}\}$
	Sparsity term (Diastole)	$\lambda_d = \{1 \times 10^{-9} \times 10^{-2}\}$

Number of filters and filter size

Table 2: Comparison of mean endpoint error varying the filters number and the filter size for a one dictionary systole and one dictionary for diastole. Notice the best result is attained with $M = 32$ and $L = 48$.

		Filter number, M				
		8	16	24	32	48
Filter size, L	8	0.1473	0.1477	0.1477	0.1475	0.1477
	16	0.1476	0.1477	0.1478	0.1478	0.1479
	24	0.1474	0.1476	0.1469	0.1470	0.1475
	32	0.1477	0.1471	0.1477	0.1469	0.1475
	40	0.1476	0.1473	0.1478	0.1482	0.1474
	48	0.1470	0.1467	0.1466	0.1465	0.1470

Parameters λ and ρ

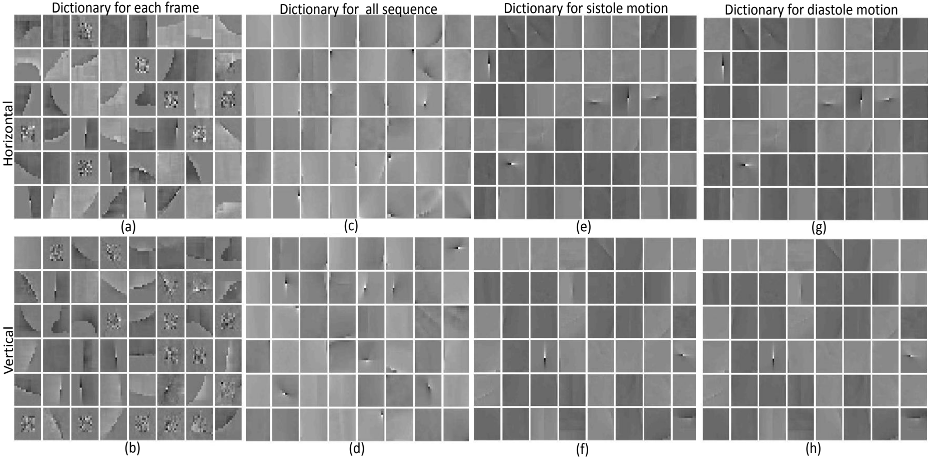
	$\rho_0 = 25$	$\rho_1 = 50$	$\rho_2 = 100$	$\rho_3 = 150$
$\lambda_0 = 0.05$	28.9992	29.2101	28.6905	**29.3802**
$\lambda_1 = 0.1$	24.9587	24.9932	25.2910	25.2395
$\lambda_2 = 0.2$	20.9748	21.1248	21.2963	21.3028
$\lambda_3 = 0.3$	18.7247	18.8688	18.8703	18.8667
$\lambda_4 = 0.5$	16.1208	16.1271	15.9824	16.0115

Table 3: Image quality (PSNR) with 100 iteration of (CBPDN) for different choices of λ and ρ by using the motion horizontal ground-truth.

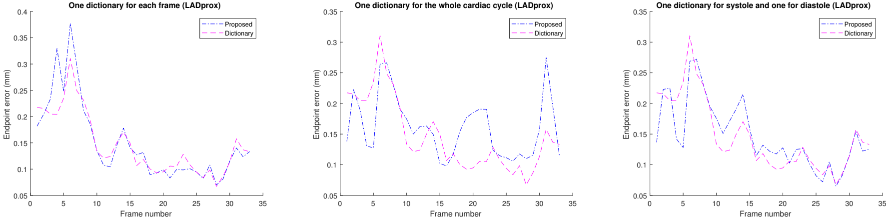
	$\rho_0 = 25$	$\rho_1 = 50$	$\rho_2 = 100$	$\rho_3 = 150$
$\lambda_0 = 0.05$	26.9361	27.2072	27.2159	**27.3542**
$\lambda_1 = 0.1$	22.8906	22.8575	22.9179	23.1302
$\lambda_2 = 0.2$	19.2978	19.2049	19.3376	19.3352
$\lambda_3 = 0.3$	17.3294	17.4144	17.4043	17.3599
$\lambda_4 = 0.5$	15.1404	15.2372	15.2145	15.1526

Image quality (PSNR) with 100 iteration of (CBPDN) for different choices of λ and ρ by using the motion vertical ground-truth.

3 dictionaries, 3 scenarios



Mean Endpoint Error

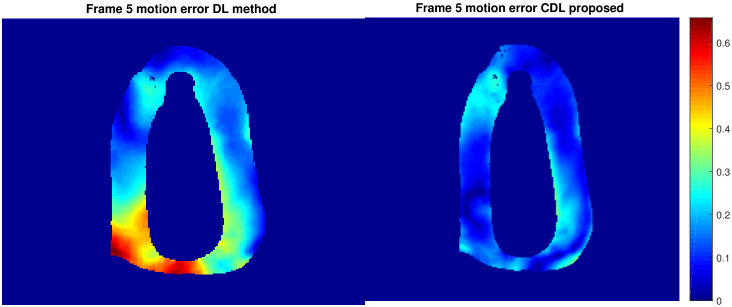


(a) (b) (c)

Mean endpoint error (in mm) for the LADprox sequence by (a) training a convolutional dictionary for each frame (error: 0.1556), (b) training a convolutional dictionary for all the frame (error: 0.1601) and (c) training two convolutional dictionaries (one for systole and one for diastole motions) (error: 0.147). The error for the method of [3][1]is 0.147.

[1] N., Ouzir, 2018.

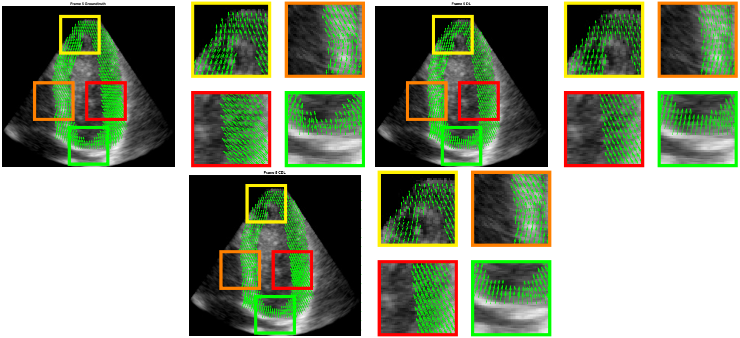
Error maps estimation



(d) (e)

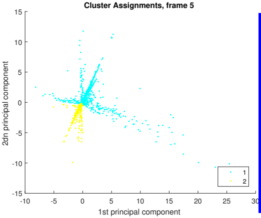
Error map for the 5th frame. Motion estimation using standard dictionary learning [3] (left) and proposed method (right).

Estimation motion comparison

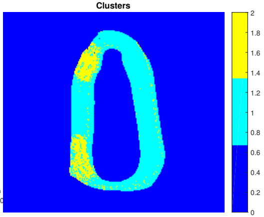


Ground-truth (top-left) and estimated meshes of the 5^{th} frame of motion estimation using standard dictionary learning [3] (top-right), and motion estimation using convolutional dictionary (bottom).

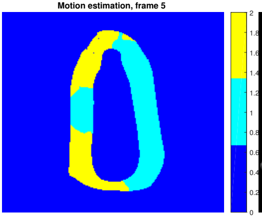
Coefficients vs motion estimation



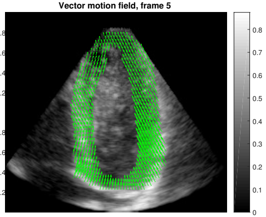
(f)



(g)



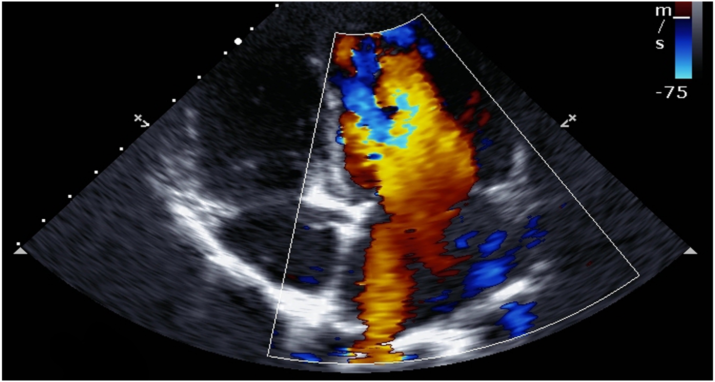
(h)



(i)

Conclusions and future work

- A new method for cardiac motion estimation in ultrasound imaging was presented.
- In the future will be studied the sparse decomposition for anomaly detection and cardiac tissue classification.



Questions?

References

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