

# Demosaicking Multispectral Images by Sphere Packing Filter Design

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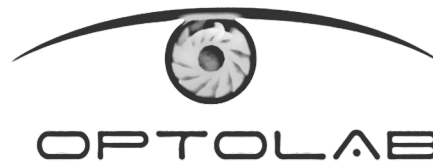
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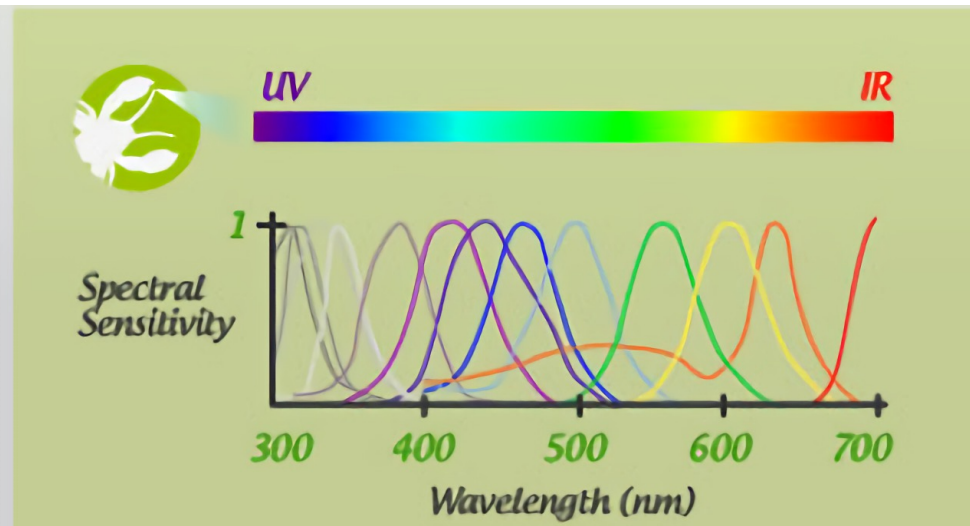
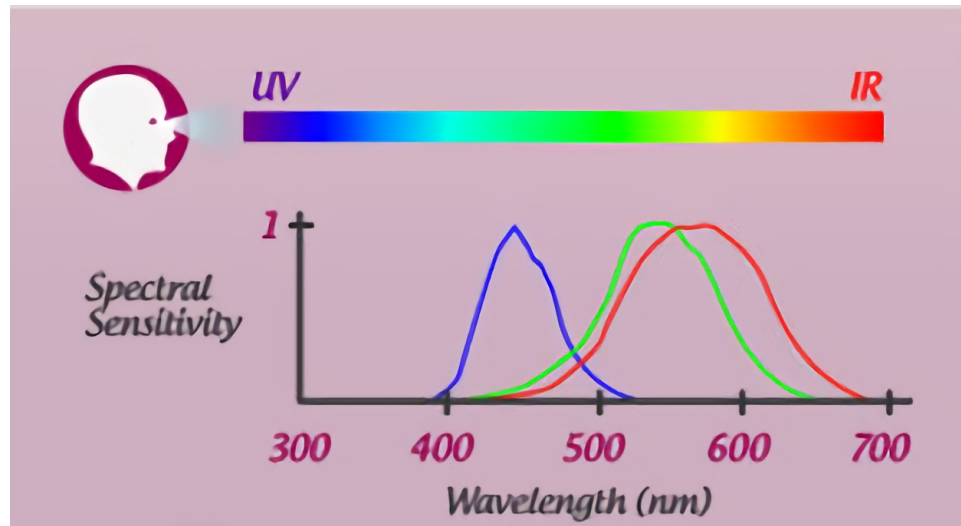


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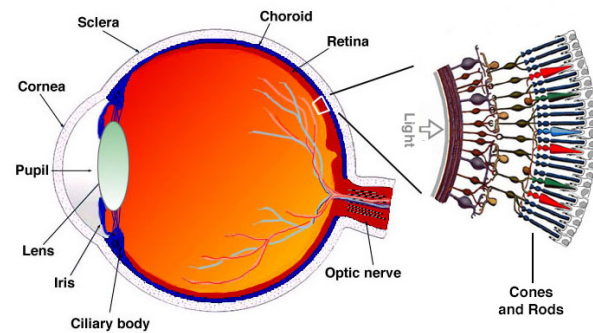
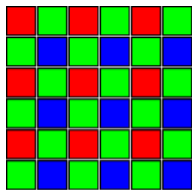


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# Motivation



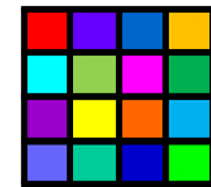
RGB



Human  
3 Different types of Cones



MSI



Mantis Shrimp  
12 Different types of Cones

# Mathematical Model

Continuous acquisition of the grayscale mosaic

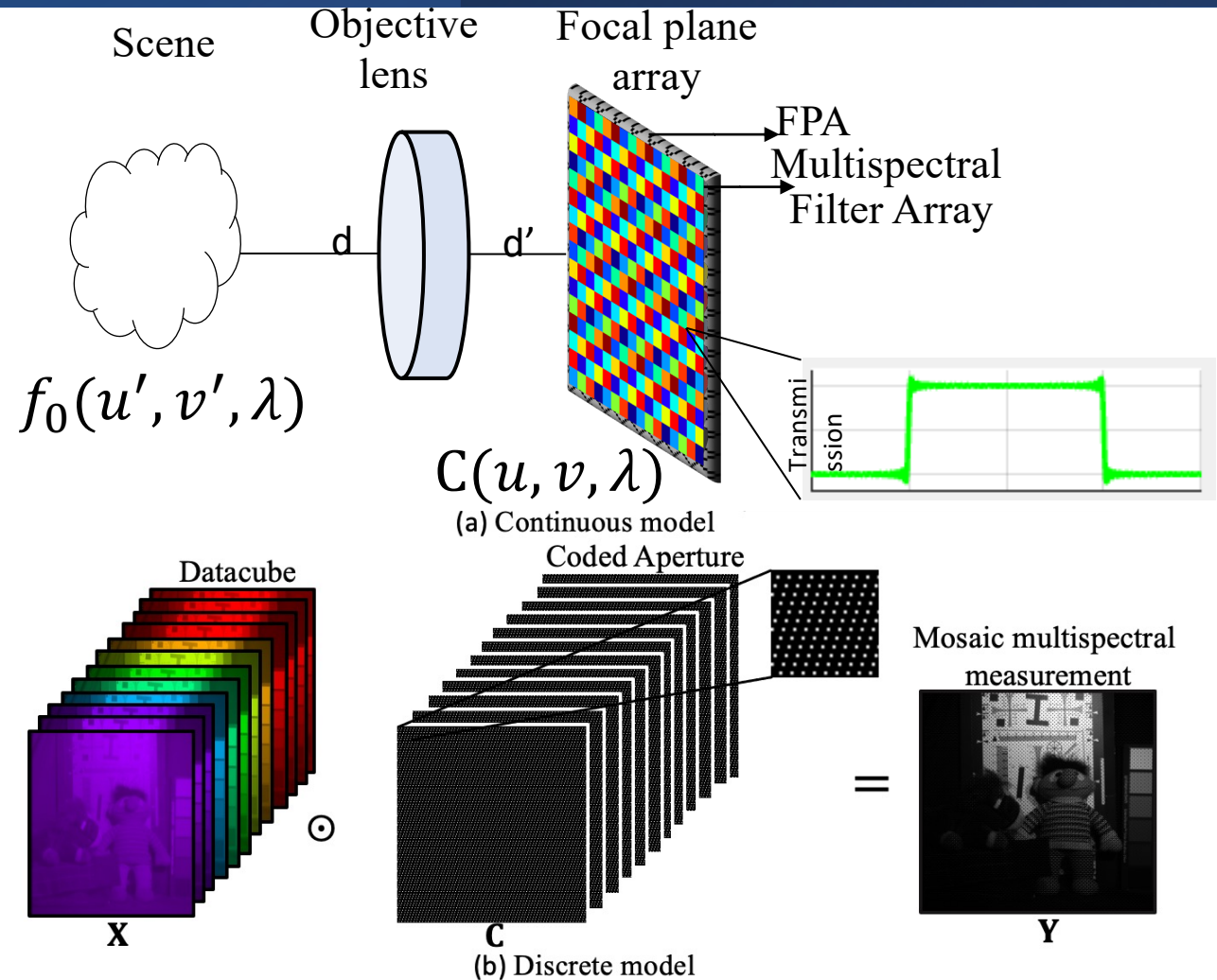
$$y(u, v) = \int_{\Lambda} C(u, v, \lambda) f_0(u', v', \lambda) d\lambda \quad (1)$$

Where  $C$  is coded aperture in focal plane and  $f_0$  is the scene.

The acquisition of the grayscale mosaic compressive multispectral (discrete) projection of  $L$  spectral bands is

$$\mathbf{Y} = \sum_{l=1}^L \mathbf{X}_l \odot \mathbf{C}_l + \Omega \quad (2)$$

Where  $\mathbf{X}_l$  the individual grayscale image in the  $l^{\text{th}}$  wavelength



# What is Sphere Packing?

2D



$$\text{Density} = \frac{\pi}{\sqrt{12}} \approx 0.9068$$

3D



$$\text{Density} = \frac{\pi}{\sqrt{18}} \approx 0.7404$$



# History of Sphere Packing Problem



- 1611
- Johannes **Kepler conjectured** about the closest packing of equal spheres.
- He did not have a prove to the conjecture.



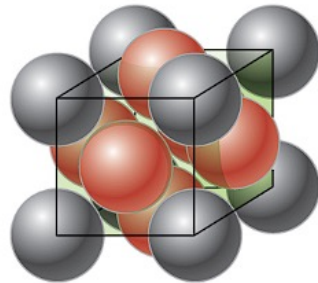
- 1998
- Thomas Hales provides the formal proof of Kepler's conjecture.
- But **eliminating all possible irregular arremgements** is very difficult, and this is what made the Kepler conjecture so hard to prove.



- 1831
- Carl Friedrich Gauss proved that the highest packing fraction that can be achieved by any packing of equal sphere.
- He proved that the Kepler conjecture is true if the spheres have to be arranged in a **regular lattice**.



- 2017
- Maryna Viazovska solved sphere packing problem in **8-dimensions** [1] (E\_8 lattice). And in collaboration with others **24-dimensions** [2] (Leech lattice).
- **Winner of the Fields Medal 2022.**



Face-centered cubic structure

**Theorem 1:** No packing of congruent balls in Euclidean three space has density greater than that of the **face-centered cubic packing**, which corresponds to:

$$\rho = \frac{\pi}{3\sqrt{2}} \approx 0.7405$$

[1] M. S. Viazovska, "The sphere packing problem in dimension 8," Annals of Mathematics, vol. 185, no. 3, pp. 991-1015, 2017.

[2] H. Cohn, A. Kumar, S. D. Miller, D. Radchenko, and M. Viazovska, "The sphere packing problem in dimension 24," Annals of Mathematics, vol. 185, no. 3, pp. 1017-1033, 2017.

# Proposed Multispectral Filter Array (MSFA)

The problem associated to the MSFA-sensing is

$$\mathbf{B} = (a \odot \mathbf{I} + b \odot \mathbf{J}) \pmod{L + 1} \quad (3)$$

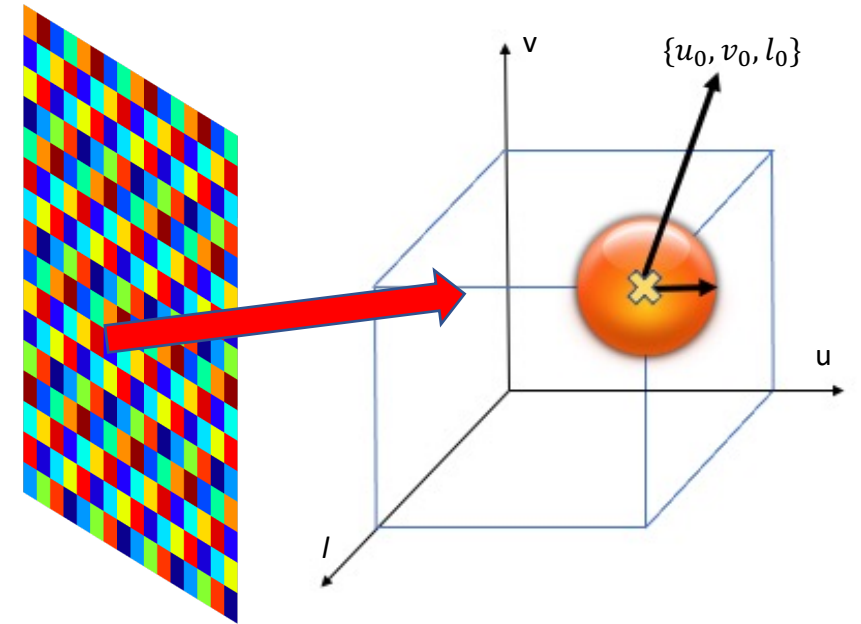
$\mathbf{I} = \mathbf{g}^T \otimes \mathbf{q}$ ,  $\mathbf{g}$  is a ones 1D-vector with length  $L$ ,  $\mathbf{q}$  is equal to  $[1, \dots, L]$  and  $\mathbf{J} = \mathbf{I}^T$ .  $a$  and  $b$  were calculated with the proposed algorithm in [3]

The positions of the MSFA-OSP are given by:

$$\mathbf{E} = \mathbf{A} \otimes \mathbf{B} \quad (4)$$

Where  $\mathbf{A}$  is a matrix of all ones such that  $\mathbf{A} \in 1^{\alpha \times \beta}$ , where  $\alpha = \lfloor \frac{M}{L} \rfloor$  and  $\beta = \lfloor \frac{N}{L} \rfloor$ ,  $M$  and  $N$  are number of pixels.

$\odot$  denotes the Hadamard product and  $\otimes$  represents the Kronecker product.

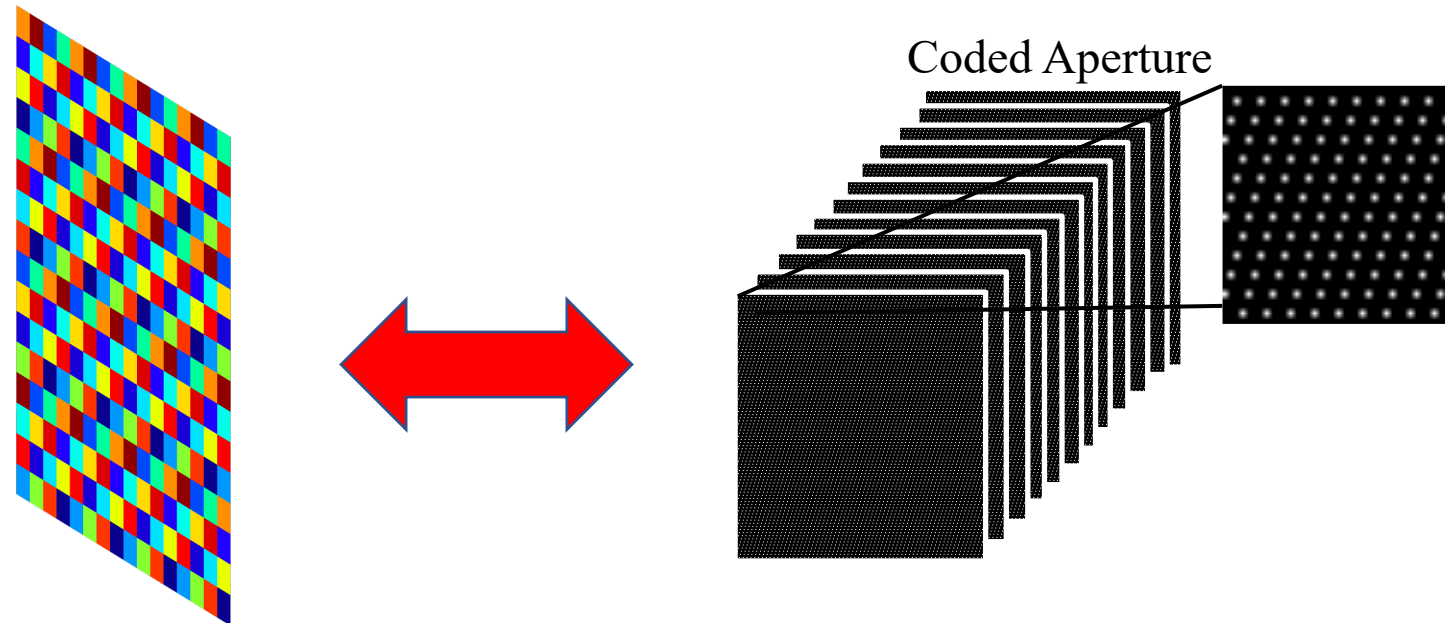


# Proposed MSFA

The resulting positions of the MSFA can be expressed in binary coded aperture form:

$$C_{m,n,l} = \begin{cases} 1 & \text{if } l = E_{m,n} \\ 0 & \text{if } l \neq E_{m,n} \end{cases} \quad (5)$$

Where  $m \in \{0, \dots, M - 1\}$ ,  $n \in \{0, \dots, N - 1\}$ ,  $l \in \{1, \dots, L\}$ .



# Sphere Packing Upper Bound

## Optimal Sphere Packing

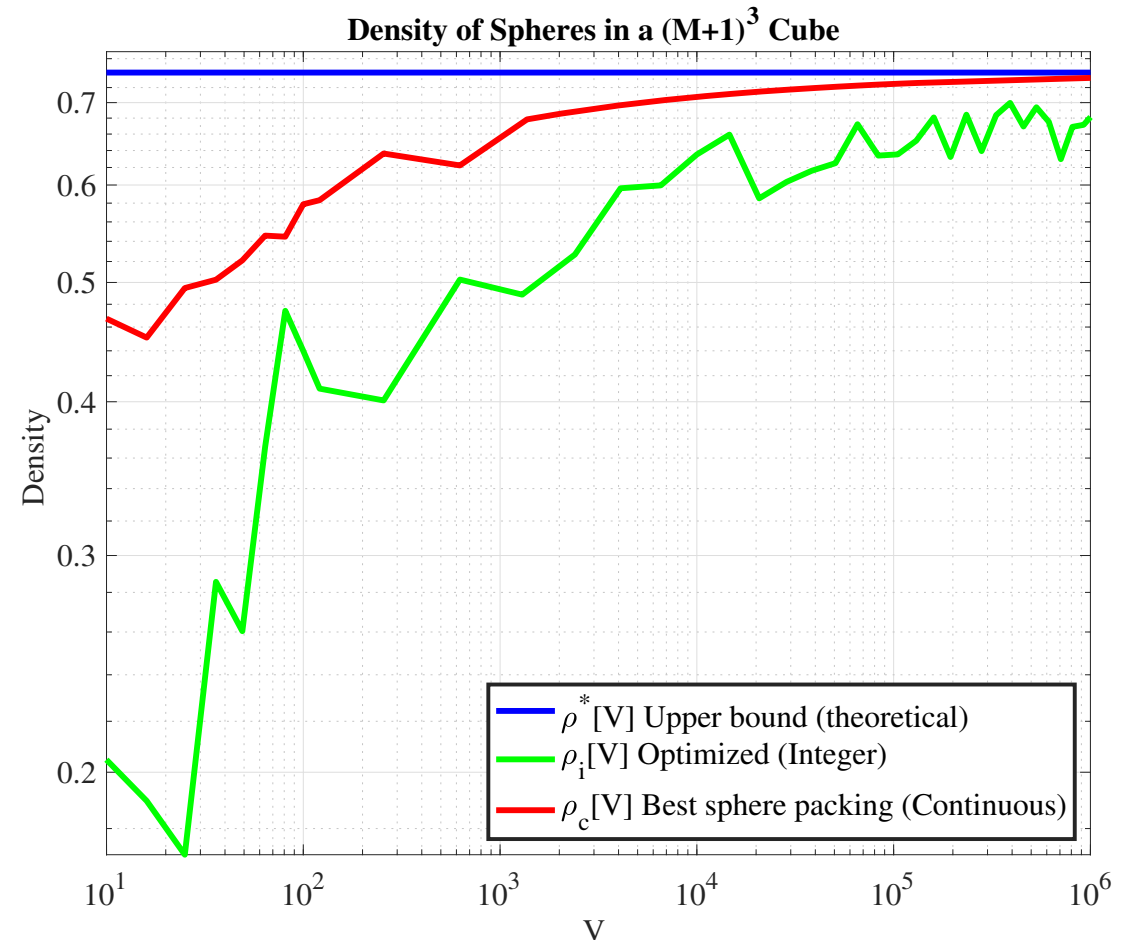
$$d^*(V) = \max\left(\min_{1 \leq k_1 < k_2 \leq V} D_{k_1, k_2}\right) \quad (6)$$

$V$  is the number of the spheres.

$D_{k_1, k_2}$  is the all pairwise distance matrix

Theoretical upper bound sphere packing density

$$\rho^*(V) = 2\sqrt[3]{\frac{(\sqrt{V} + 1)^3}{4V\sqrt{2}}} \quad (7)$$

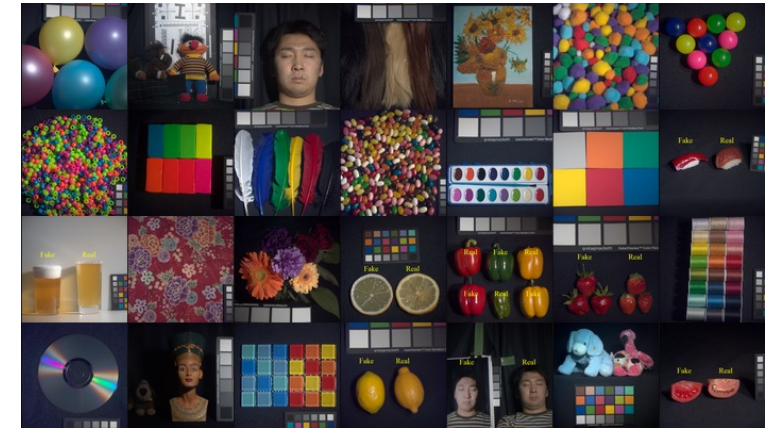


Continuous model: O. Packomania, "Packings of equal spheres in fixed-sized containers with maximum packing density," URL <http://www.packomania.com>, 2013.



# Dataset and Network

- Use Cave Dataset [4]
  - 32 Scenes with 31 spectral bands and 512x512 pixels.
  - Scenes resized to 256x256 pixels and 16 spectral bands.
- State-Of-The-Art in Demosaicking Algorithms:
  - **WB**: Weighted bilinear [5]
  - **itID**: Iterative intensity difference [6]
  - **itNCD**: Iterative nearby channel difference [6]
- TRevSCI-net [7] (3D-CNN for tensor completion) was training with 10560 cubes.
  - Name: Tensor reversible snapshot compressive imaging.
  - 80% train and 20% validation.
  - L1 cost function



[4] F. Yasuma, T. Mitsunaga, D. Iso, and S. K. Nayar, “Generalized assorted pixel camera: Postcapture control of resolution, dynamic range, and spectrum”, IEEE Transactions on Image Processing, 2010.

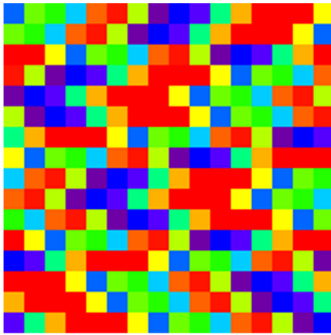
[5] J. Brauers and T. Aach, “A color filter array based multispectral camera”, Workshop Farbbildverarbeitung, Oct 2006.

[6] S. Mihoubi, O. Losson, B. Mathon, and L. Macaire, “Multispectral demosaicing using intensity in edge-sensing and iterative difference-based methods”, in 2016 12th International Conference on Signal-Image Technology Internet-Based Systems (SITIS), 2016.

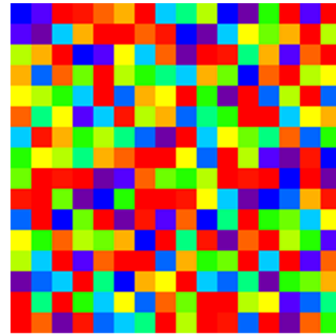
[8] Z. Cheng, B. Chen, G. Liu, H. Zhang, R. Lu, Z. Wang, and X. Yuan, “Memory-Efficient Network for Large-scale Video Compressive Sensing”, Proceedings of the IEEE Computer Society Conference on CVPR, 2021.

# MSFA patterns

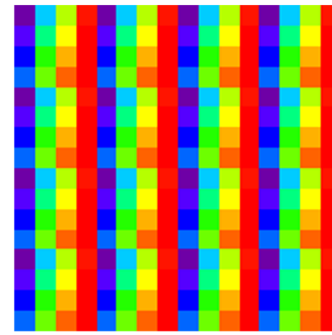
Random



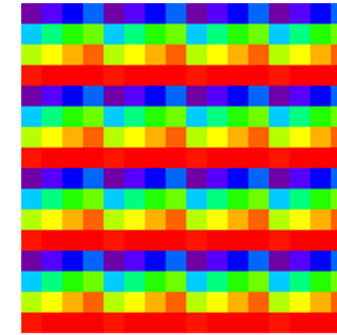
Blue Noise<sup>[8]</sup>



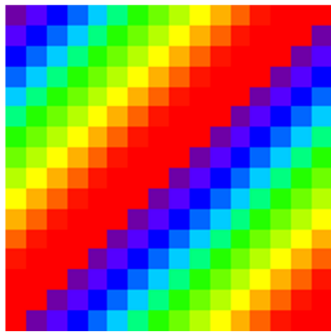
Brauers<sup>[9]</sup>



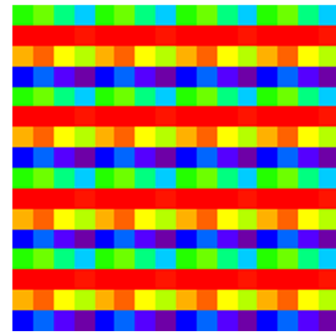
Sequential



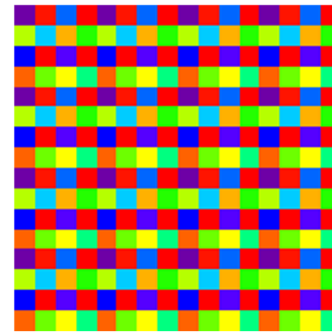
Uniform



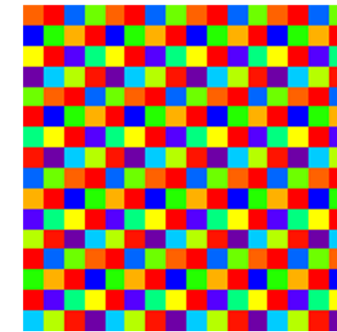
IMEC<sup>[10]</sup>



BTES<sup>[11]</sup>



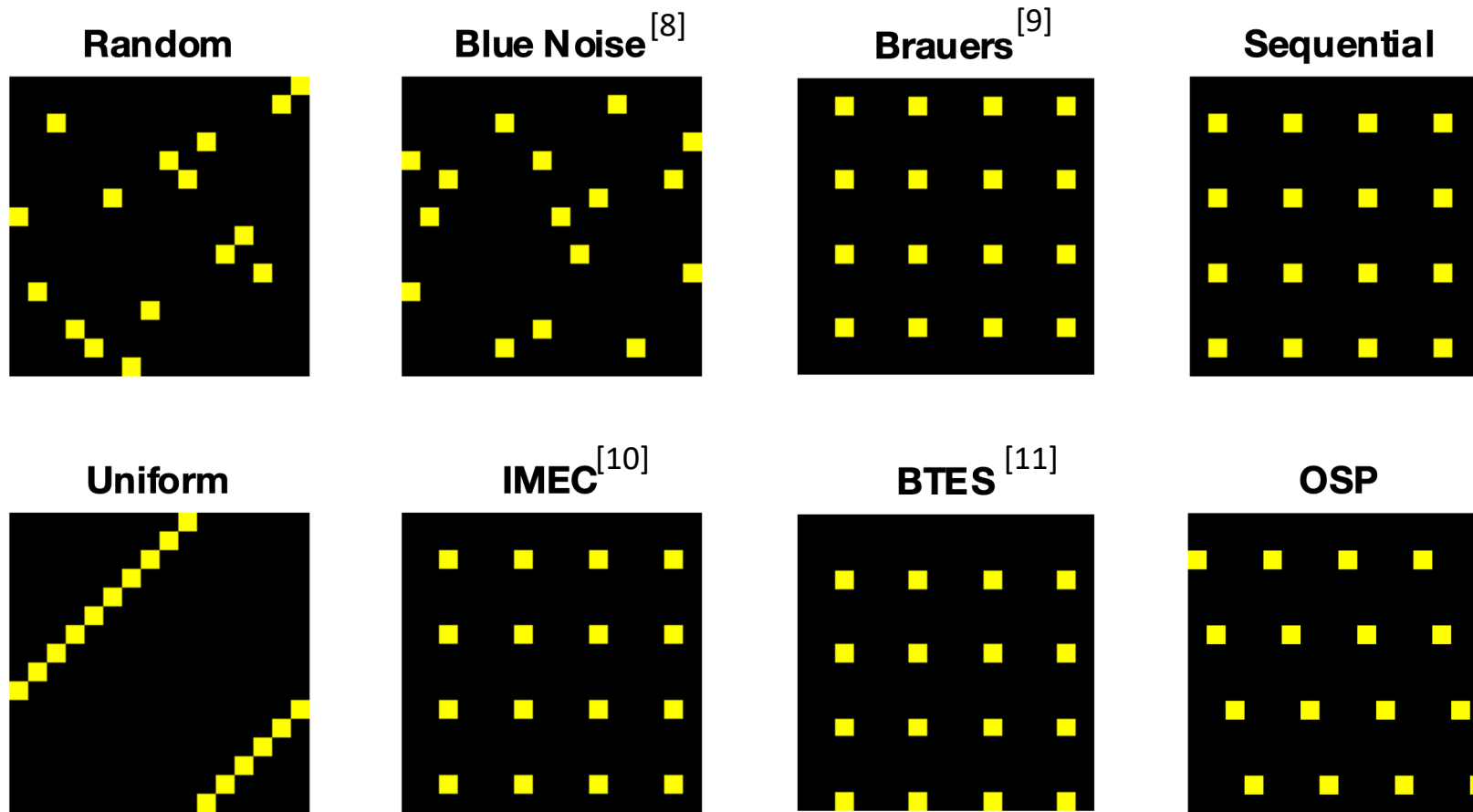
OSP



\*Blue Noise is the only irregular lattice included in the comparison but satisfies the restricted isometry property (RIP).

[8] C. V. Correa, H. Arguello, and G. R. Arce, "Spatiotemporal blue noise coded aperture design for multi-shot compressive spectral imaging", Journal of the Optical Society of America A, Dec 2016.  
[9] J. Brauers and T. Aach, "A color filter array based multispectral camera", Workshop Farbbildverarbeitung, Oct 2006.  
[10] B. Geelen, N. Tack, and A. Lambrechts, "A compact snapshot multispectral imager with a monolithically integrated per-pixel filter mosaic", Advanced Fabrication Technologies for Micro/Nano Optics and Photonics VII, 2014.  
[11] L. Miao, H. Qi, R. Ramanath, and W. Snyder, "Binary tree-based generic demosaicking algorithm for multispectral filter arrays", IEEE Transactions on Image Processing, 2006.

# MSFA patterns

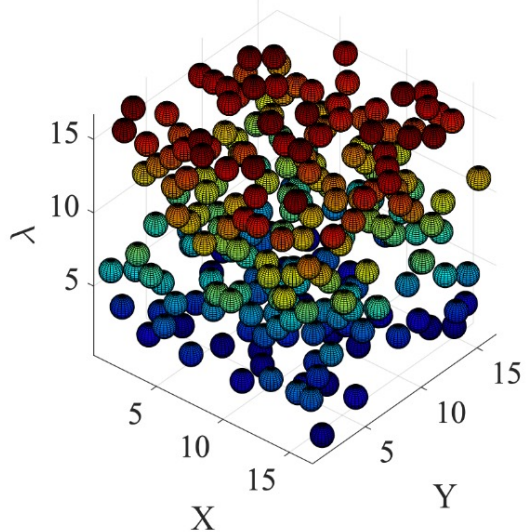


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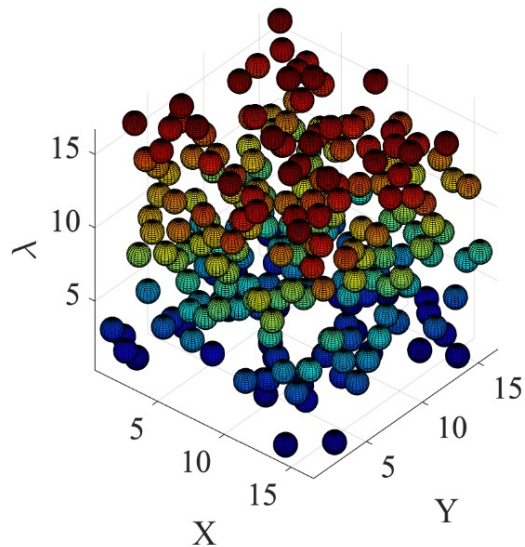
[8] C. V. Correa, H. Arguello, and G. R. Arce, "Spatiotemporal blue noise coded aperture design for multi-shot compressive spectral imaging", Journal of the Optical Society of America A, Dec 2016.  
[9] J. Brauers and T. Aach, "A color filter array based multispectral camera", Workshop Farbbildverarbeitung, Oct 2006.  
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[11] L. Miao, H. Qi, R. Ramanath, and W. Snyder, "Binary tree-based generic demosaicking algorithm for multispectral filter arrays", IEEE Transactions on Image Processing, 2006.

# Density Comparison

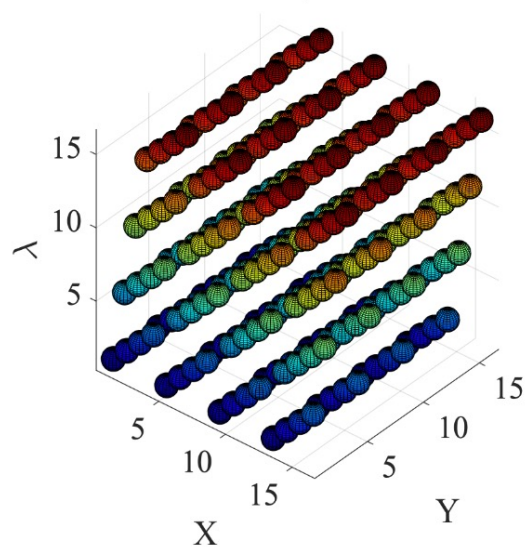
**Random**  
 $d=1.41$   $\rho=0.08$



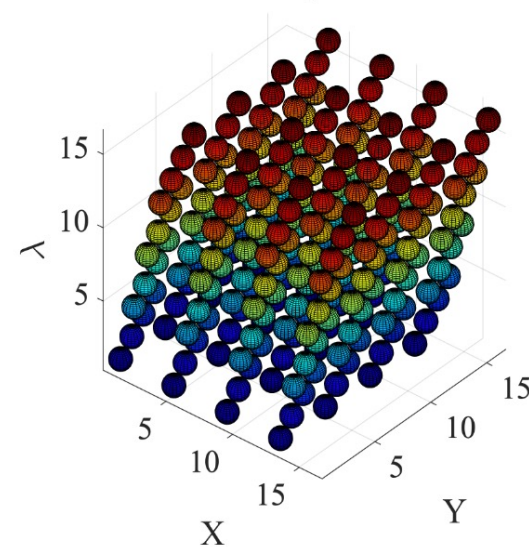
**BN**  
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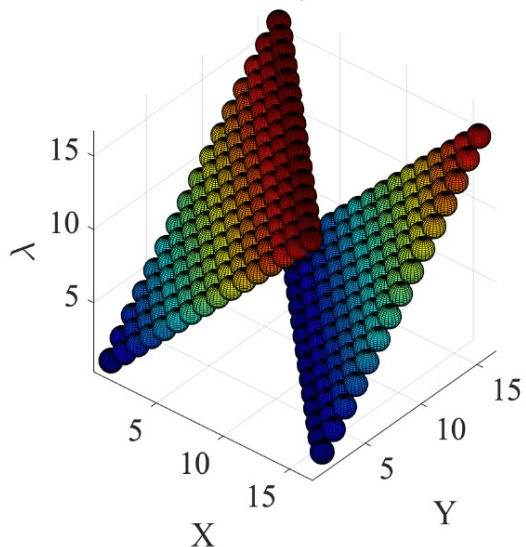
**Brauers**  
 $d=1.41$   $\rho=0.08$



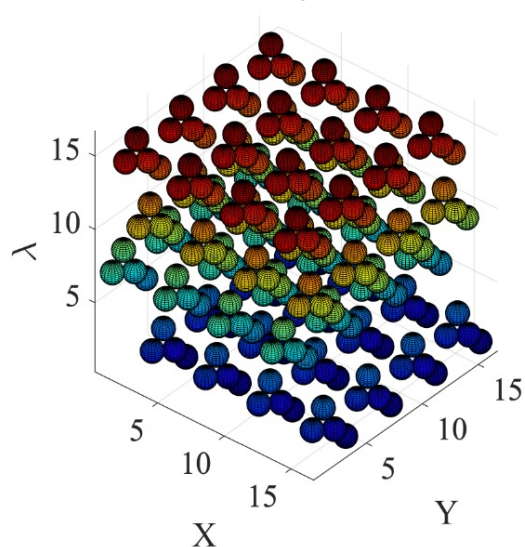
**Seq**  
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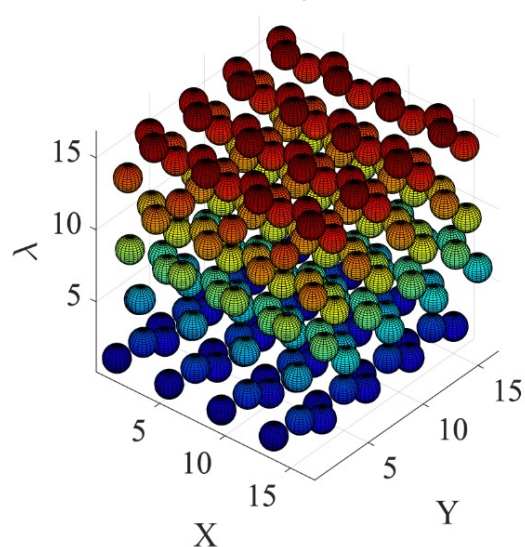
**Uniform**  
 $d=1.41$   $\rho=0.08$



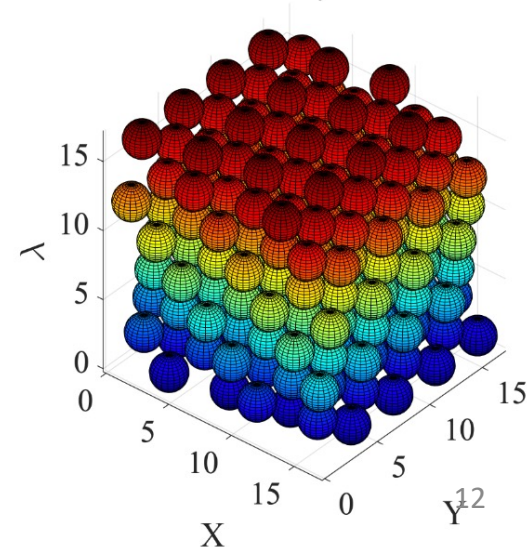
**IMEC**  
 $d=1.41$   $\rho=0.08$



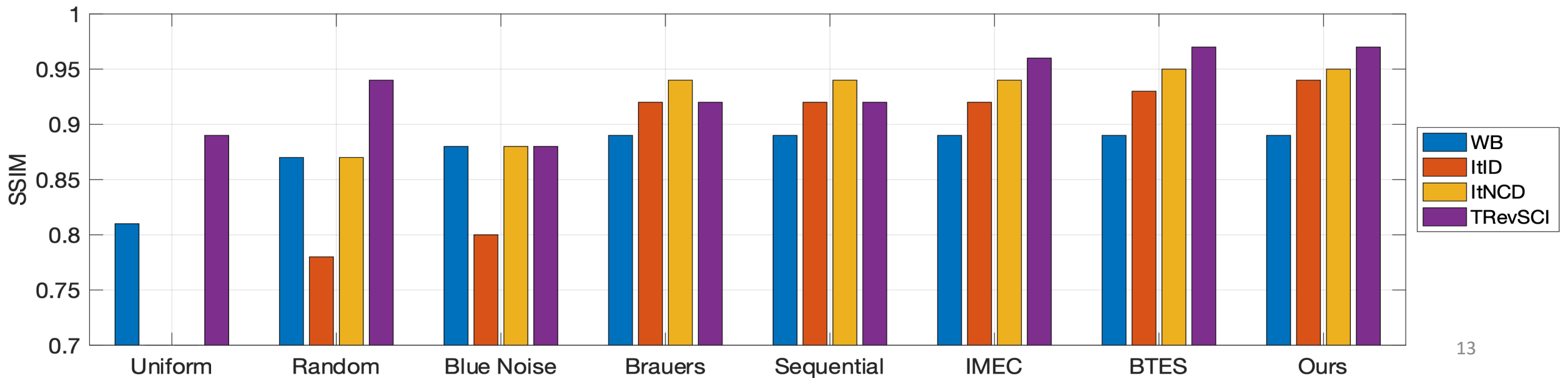
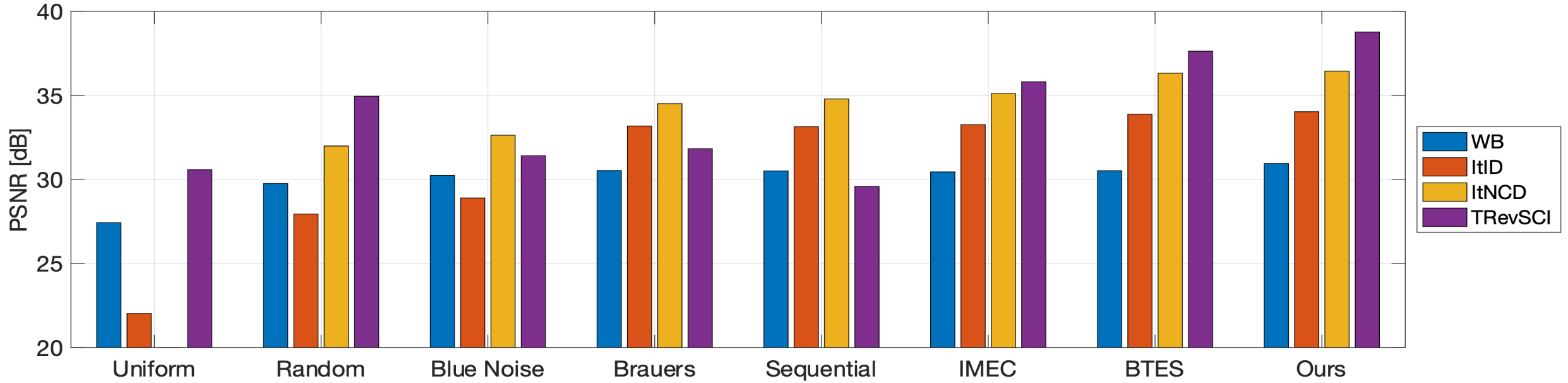
**BTES**  
 $d=1.73$   $\rho=0.14$



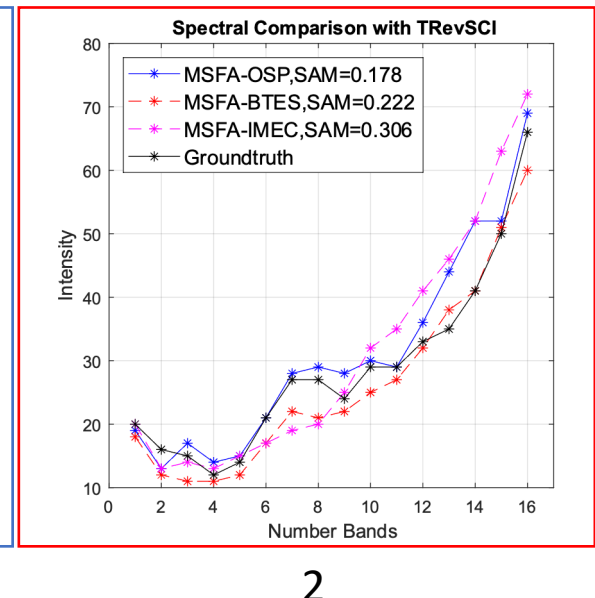
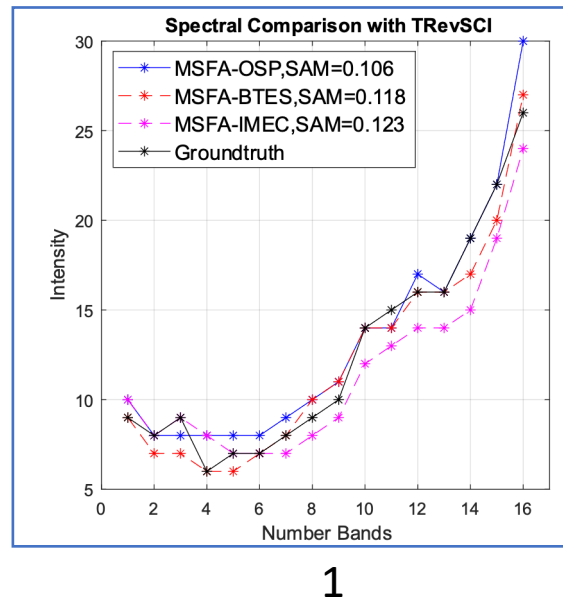
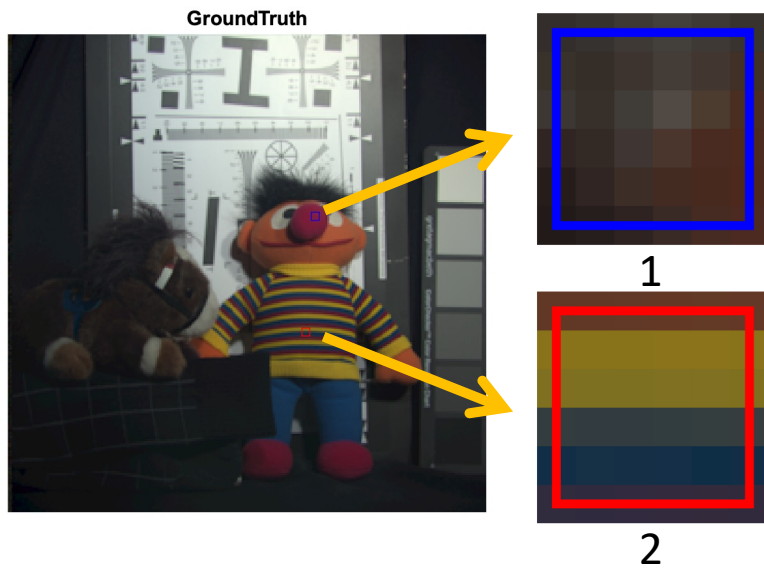
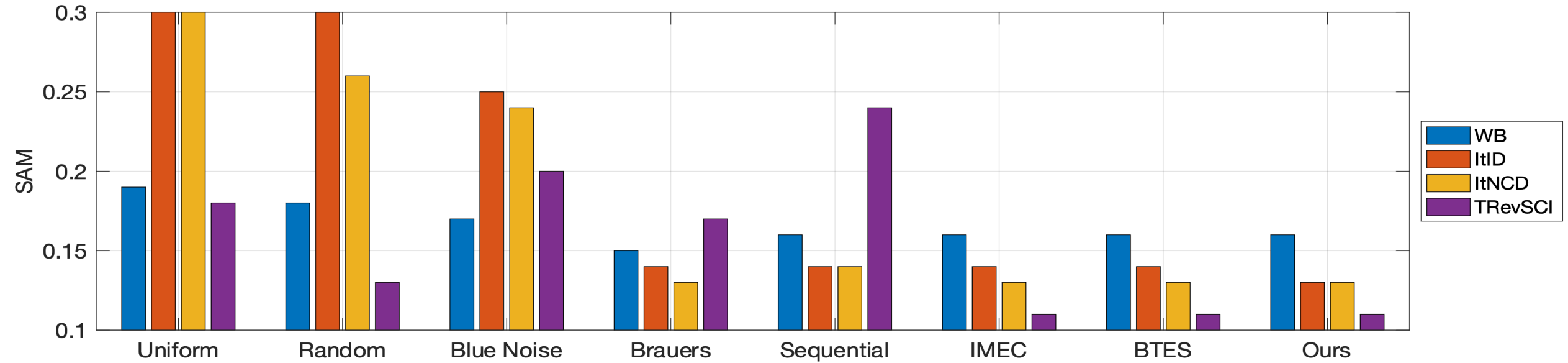
**OSP**  
 $d=2.45$   $\rho=0.4$



# Results: Cave Dataset



# Results: Cave Dataset



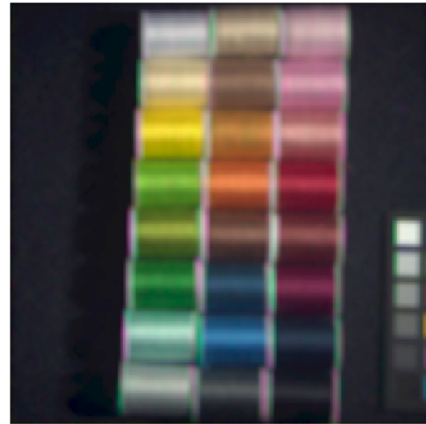
# Comparison: RGB

MSFA-BTES

Groundtruth



WB, PSNR=28.7305dB



ItID, PSNR=31.9678dB



ItNCD, PSNR=35.6258dB



TRevSCI, PSNR=36.7176dB



Groundtruth



WB, PSNR=29.2541dB



ItID, PSNR=32.486dB



ItNCD, PSNR=35.7623dB



TRevSCI, PSNR=39.3015dB



MSFA-OSP

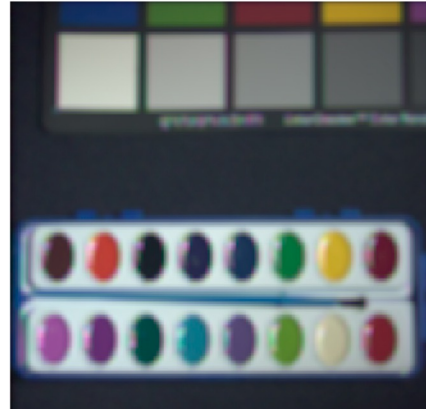
# Comparison: RGB

MSFA-BTES

Groundtruth



WB, PSNR=24.4998dB



ItID, PSNR=27.9234dB



ItNCD, PSNR=30.1798dB



TRevSCI, PSNR=31.6954dB

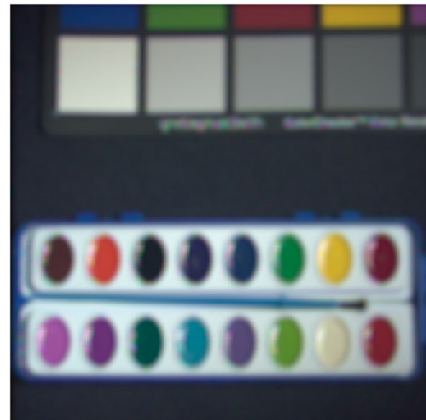


MSFA-OSP

Groundtruth



WB, PSNR=24.665dB



ItID, PSNR=28.1947dB



ItNCD, PSNR=30.4826dB

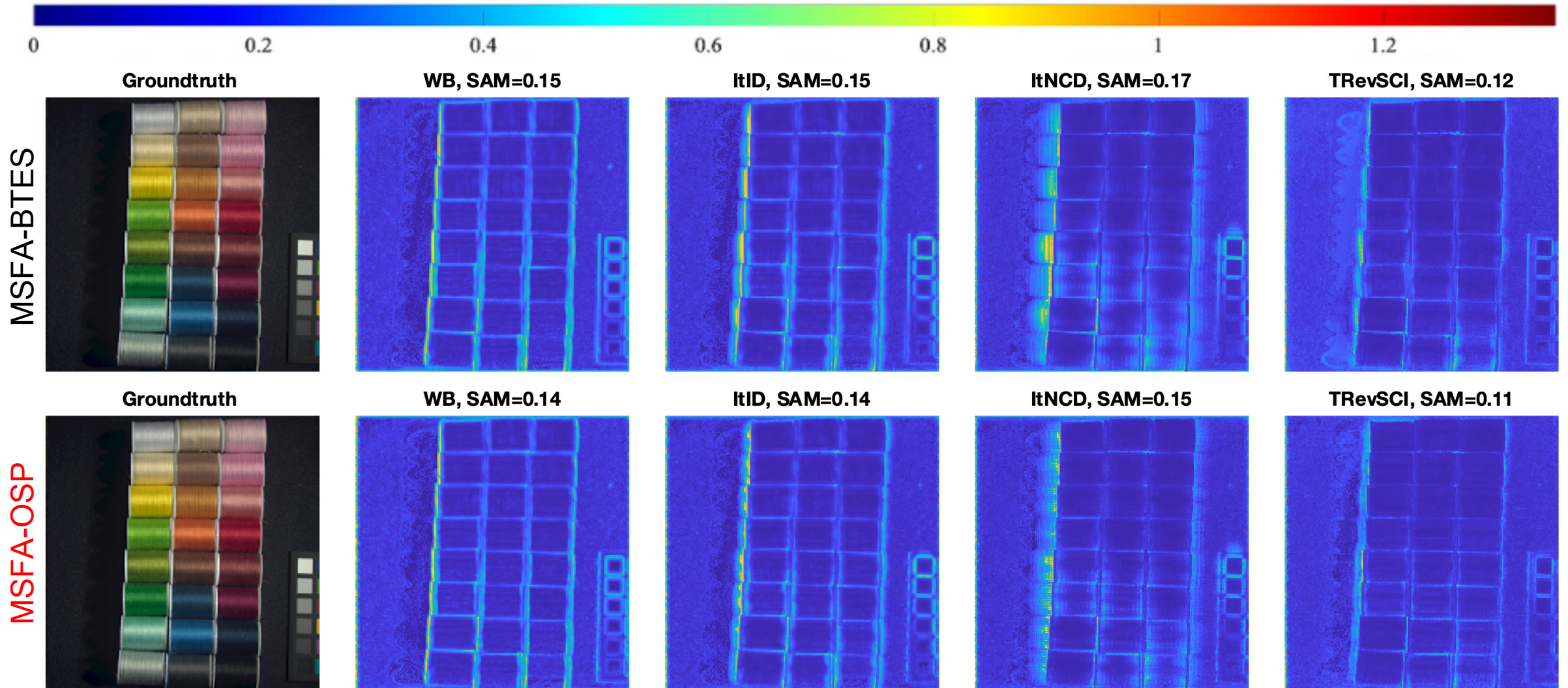


TRevSCI, PSNR=33.8975dB

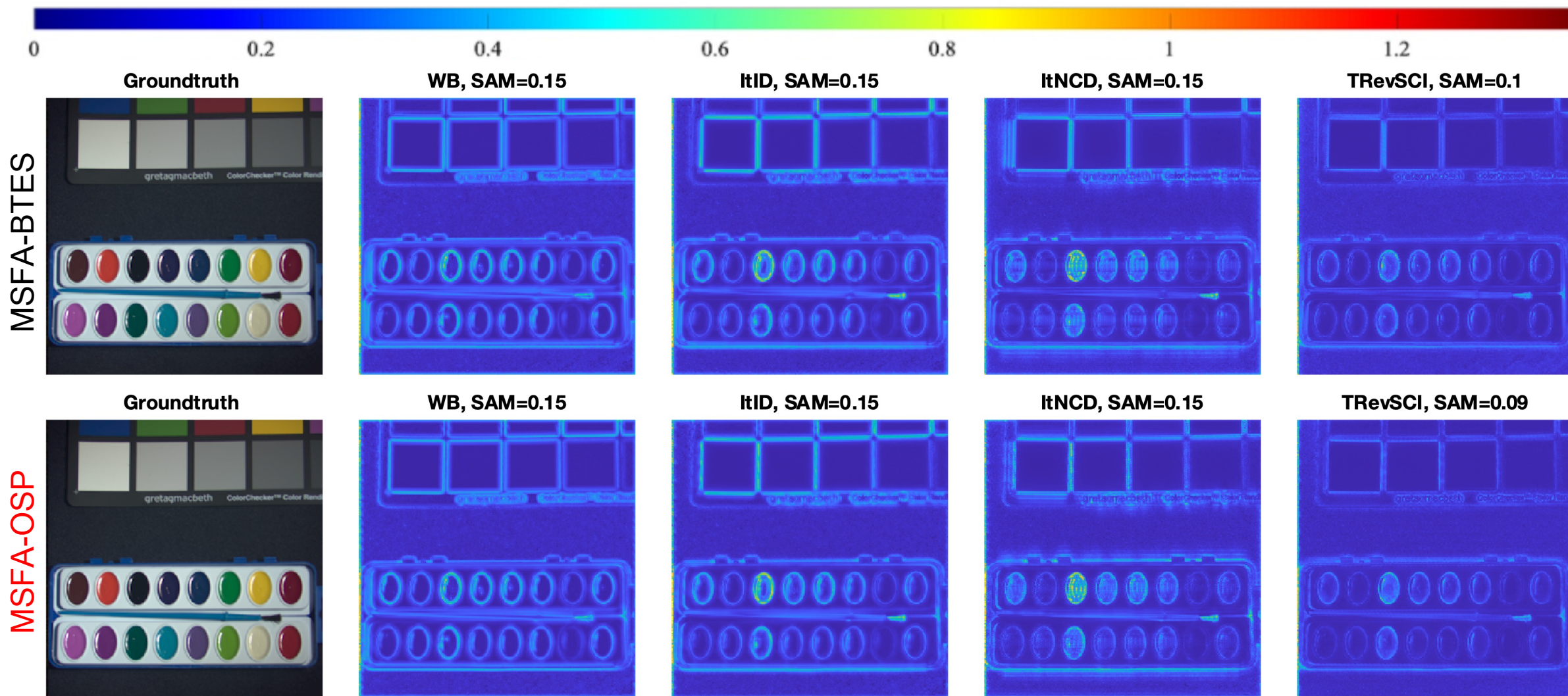




# Comparison: SAM



# Comparison: SAM

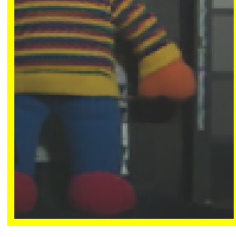
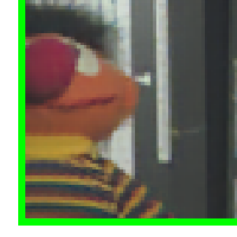
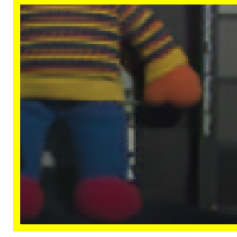
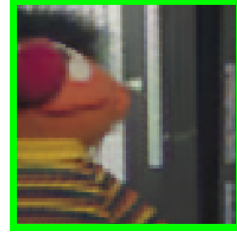
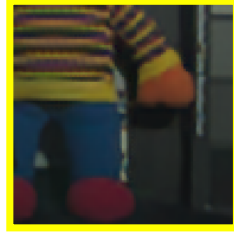
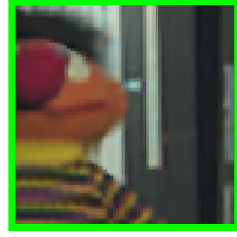
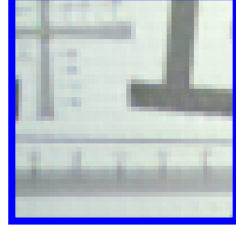
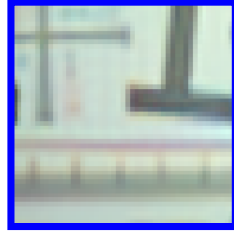
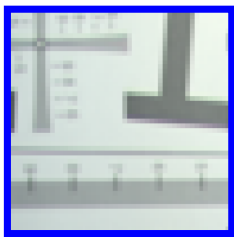


# Comparative with TRevSCI

- Aliasing
- Zipper effect
- Color artifacts



Groundtruth



Groundtruth

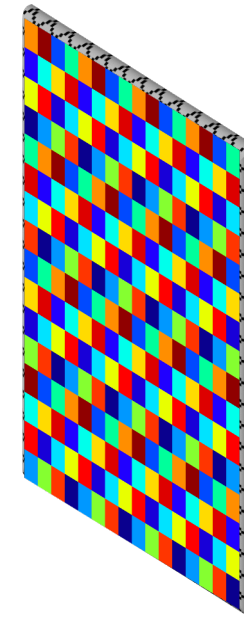
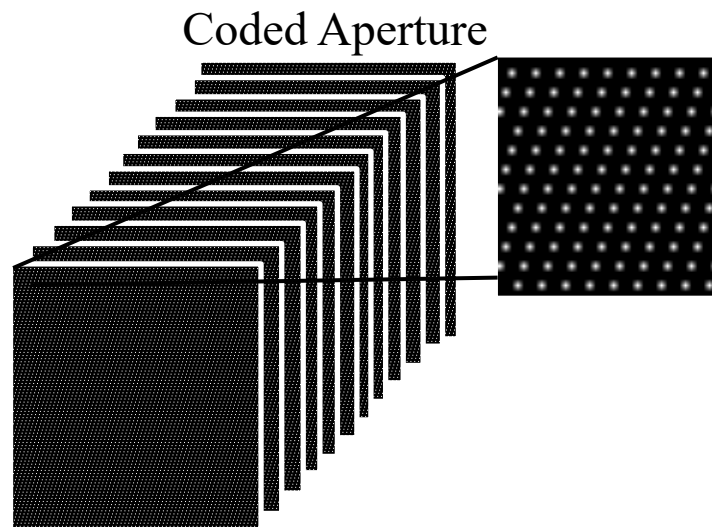
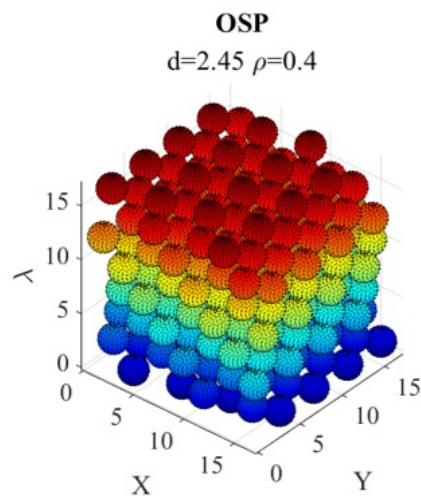
IMEC16

BTES16

OSP16

# Conclusion

- We present a Multispectral Filter Array by Optimal Sphere Packing (MSFA-OSP). This approach extends the idea of CFA (RGB) to multispectral imaging.
- Our MSFA-OSP provides 2 [dB] extra of PSNR compared to the best of other SOTA MSFA.
- The advantages of the optimal filter distribution include reducing artifacts such as false colors and the zipper effect of demosaicking algorithms.
- Future works will extend the sphere packing framework to higher dimensions of the plenoptic function, such as compressive spectral-video.





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