A Sphere Packing Approach to Design Multiplexed Multispectral Filter Arrays

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Abstract: We propose a method to design a multiplexed Multispectral Filter Array (MSFA) based on Optimal Sphere Packing (OSP) in a 4D-Euclidean space that promotes uniformity and improved signal-to-noise ratio in the measurement process of the 3D-datacube while delivering enhanced reconstructions.

1. Introduction

Multispectral images represent spatio-spectral information in three-dimensions, known as datacubes. Nevertheless, the high costs of available multispectral cameras limit their widespread usage. In contrast, digital cameras with three color bands, based on the color filter array (CFA), have been used to simplify the acquisition of color images, being the most relevant CFA the Bayer filter [1]. An extension of the CFA concept is the Multispectral Filter Array (MSFA), which allows the acquisition of spectral features for enhanced discrimination, although the quality of the reconstructed 3D datacubes from a single 2D snapshot heavily depends on the design of the MSFA.

A well-designed MSFA [2, 3] improves the spectral resolution without heavy losses in the spatial resolution. For instance, our design by OSP [4, 5] places narrow spectral filters reducing effects such as the zipper effect and aliasing. Although the SP approach provides with a method to design MSFAs without limiting the number of filters, the main problem for any design of narrow filter MSFAs is the severe subsampling of the measurements and the reduced signal to noise ratio. In this work, we propose a method to design multiplexed MSFAs by extending the OSP approach, by increasing the number of bands sampled per pixel, allowing a better signal to noise in the measurements. Therefore, we extend the OSP problem from the original 3D-SP towards a 4D-SP problem, where the new dimension is not a physical dimension but provides an additional degree of freedom to properly arrange the multiplexed filters.

2. Methods

The acquisition of the grayscale mosaic of compressive multiplexed multispectral projections of F spectral bands with M degrees of multiplexing is given by

$$\mathbf{Y} = \sum_{m=1}^{M} \sum_{l=1}^{F} \mathcal{X}_{(:,:,l,m)} \odot C_{(:,:,l,m)} + \mathbf{\Omega}, \tag{1}$$

where $\mathcal{X}_{(:,:,l,m)} \in \mathbf{R}^{N_x \times N_y}$ is the l^{th} spectral band with $N_x \times N_y$ number of pixels, $C_{(:,:,l,m)} \in \mathbf{R}^{N_x \times N_y}$ denote the positions of the filters at l^{th} wavelength and m^{th} multiplexing of the filters, $\odot$ is the Hadamard product, and $\mathbf{\Omega}$ is the Gaussian noise. The proposed design for the MSFA is equivalent to packing spheres in a 4D container. The 4D multiplexed MSFA sensing model can be expressed as:

$$\mathcal{B}(:,:,m) = ((\alpha \odot \mathbf{V} + \beta \odot \mathbf{H} + \gamma \odot q_m) \mod F + 1), \tag{2}$$

where F is the number of bands, the $\mathbf{V}$ and $\mathbf{H}$ are the vertical and horizontal translation matrices, respectively. The vertical translation matrix is denoted by $\mathbf{V} = \mathbf{w}^T \otimes \mathbf{q}$, where $\mathbf{V} \in \mathbb{N}^{F \times F}$, $\mathbf{w}$ is a vector of all ones given by $\mathbf{w} \in \{1\}^F$, and $\mathbf{q} = [1, \ldots, F]^T$ where $\mathbf{q} \in \mathbb{N}^F$, and $\odot$ is the Hadamard product. The horizontal translation matrix is $\mathbf{H} = \mathbf{V}^T$, and the matrix $\mathbf{1} \in \mathbb{N}^{F \times F}$. The positions of MSFA at the m^{th} degree of multiplexing is given by

$$C_{(i,j,l,m)} = \begin{cases} 1 & \text{if } l = \mathcal{B}_{(i,j,m)} \\ 0 & \text{if } l \neq \mathcal{B}_{(i,j,m)}, \end{cases}$$

where indexes $i, j \in \{1, \ldots, F\}$ and $L = F^3$ is the number of spheres. Thus, the distance function of L spheres is

$$d^*(L) = \max(\min_{1 \le l_1 < l_2 \le L}, D_{l_1, l_2}), \tag{3}$$

where $D_{l_1,l_2} = \|\mathbf{p}_{l_1} - \mathbf{p}_{l_2}\|_2^2$ is the all pairwise distance matrix, $l_1, l_2 \in \{0, \ldots, L-1\}$, $\mathbf{p}$ are the centers of the spheres and index the ${l_1}^{\text{th}}$, and ${l_2}^{\text{th}}$ spheres.

3. Results

To compare the performance of the proposed MSFA design, we used the CAVE dataset [6], with spatial resolution 256×256 pixels and 16 bands for our experiment. The evaluation includes a comparison of the proposed MSFA with a random design. Fig. 1 compares the RGB groundtruth (GT) and the MSFA reconstructions of 2 and 3 multiplexed (MUX) filters, using a state-of-the-art neural network reconstruction.

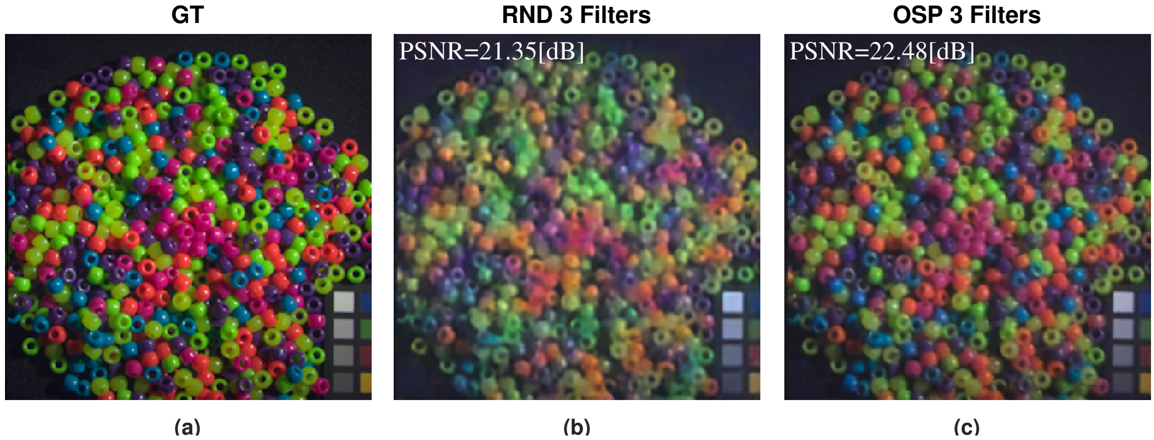

Fig. 1: (a) Groundtruth, (b) random with 3 MUX filters, (c) OSP with 3 MUX filters

Furthermore, we test two reconstructions methods. The first is GAP-TV [7] and the second is a neural network [8] developed for compressive imaging, adapted to solve our MSFA reconstruction problem. This neural net consists of an autoencoder based on U-net and implements 3D-Convolution blocks as layers, and it was trained using the CAVE dataset. The average peak-signal-to-noise ratio (PSNR)s for all dataset achieved by the MSFAs are depicted in Table 1, where the advantages of our novel MSFA for enhanced reconstructions are even more evident.

Algorithms	Random 2 MUX Filters	OSP 2 MUX Filters	Random 3 MUX Filters	OSP 3 MUX Filters
GAP-TV	27.68[dB]	28.26 [dB]	28.37[dB]	30.24[dB]
Neural Network	30.55[dB]	30.83[dB]	31.54[dB]	35.26[dB]

Table 1: PSNR for preliminary results using GAP-TV and Neural Network.

4. Conclusions

We designed a novel method to design multiplexed MSFAs by using an optimal sphere packing approach, improving the spatio-spectral sampling and the signal-to-noise ratio of the measured multiplexed datacube. Reconstruction results show promising results either using traditional reconstruction methods such as GAP-TV or novel deep neural networks. We are working on the experimental demonstration of our new multiplexed MSFAs.

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